$$A_{yy}{}^{*}F_{,xxxx} - B_{yx}{}^{*}w_{,xxxx} = 0 \qquad (17)$$

where

$$F_{,yy} = N_0 = \text{const} \qquad (18)$$

and $A_{yy}{}^{*}$, $B_{yx}{}^{*}$ are given by Eqs. (18) of Ref. 2.

Reducing Eqs. (16) and (17) to a single equation in terms of w, one gets

$$(A_{yy}{}^{*})^{-1}(B_{yx}{}^{*})^{2}\, w_{,xxxx} = p + N_0 w_{,xx} \qquad (19)$$

which resembles Eq. (a) on p. 7 of Ref. 5 for finite cylindrical bending of homogeneous plates.

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Comments

Comment on "Characteristics of the Arc in a Gerdien-Type Plasma Generator"

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In a recent technical note,[1] the authors have shown that in a Gerdien-type plasma generator exhausting into the atmosphere, where argon is the working fluid, the arc process is cyclic, and the observed flame consists of the envelope of many blown arc current paths moving about at high frequency. Measurements using equilibrium radiation concepts and based on the assumption of an ionized, current-free gas stream would be in error when applied to this stream. To provide a source of ionized gas free of current paths, the stream was expanded into a low pressure test cell (≈ 0.5 mm Hg) and the expanded flame carefully examined to determine whether the stream was influenced by the same blown arc phenomena. The same type of probe experiments as for the atmospheric case were performed.

The expanded luminous gas stream is shown in Fig. 1. The very bright section at the orifice exit contains the many blown arcs and is about the same size as the radiating jet at atmospheric pressure. The expanded flame does not contain the current paths, and electron velocity distributions found in the expanded region by Langmuir probes were Maxwellian. The expanded flame should, therefore, provide an acceptable source for further study of measurement techniques.

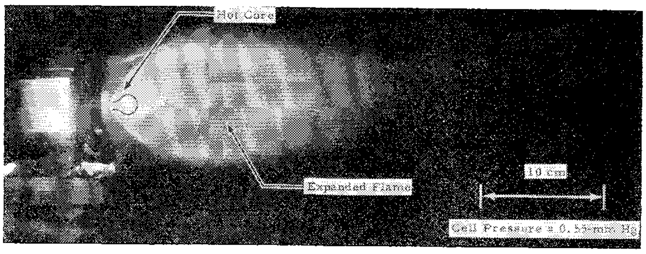

Fig. 1 Plasma generator exhausting to low pressure, showing hot core blown arc region and current free expanded flame

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[1] Dooley, M. T., McGregor, W. K., and Brewer, L. E., "Characteristics of the arc in a Gerdien-type plasma generator," ARS J. 32, 1392–1394 (1962).

Comments on "Treatment of Partial Equilibrium in Chemically Reacting Flow"

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Libby[1] has presented a method for dealing with chemically reacting flows when approximate local equilibrium is maintained among some but not all of the chemical species present. The purpose of the present comments is to point out that the solution presented may be indeterminate in some cases.

The difficulty lies in the fact that the G_j are not all independent. The relationship between the G_j is most clearly seen if the *N-L* independent G_j are chosen as those for the reactions by which species 1, . . ., *N-L* are formed from the elements. This set of G_j must be independent, since each contains only one of the Y_i, $i = 1, \ldots, N\text{-}L$. It then can be shown that

$$1 - G_i = \prod_{j=1}^{N-L} (1 - G_j)^{\nu_{ij}' - \nu_{ij}''} \qquad i = N\text{-}L + 1, \ldots, K \qquad (1)$$

The σ_j, which are functions of the rate constants, are all independent. Thus, in general, Eqs. (6) in footnote 1 will not all be linearly independent, and the M indeterminate σ_j cannot be found. This problem will not occur in all cases. If the reaction mechanism involves only *N-L* reactions, all of the G_j will be independent. Similarly if the species that are assumed to be in local equilibrium are each involved in only one reaction, the system of Eqs. (6) will be linearly independent. However, most real chemical mechanisms, particularly at high temperatures, involve many more than *N-L* reactions, and in these cases it is unlikely that a solution can be found.

Physically, this problem arises because once equilibrium is assumed it is no longer meaningful to compute contributions to gradients in species concentrations due to individual reactions. Once a species is assumed to be in local chemical equilibrium with some other species, its concentration can be

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[1] Libby, P. A., "Treatment of partial equilibrium in chemically reacting flows," ARS J. 32, 1090–1091 (1962).

determined without reference to the chemical kinetics. Mixing of equilibrium and nonequilibrium equations is valid if the chemical system is treated as two subsystems, one of which is assumed to be in local chemical equilibrium and the other rate-controlled. The systems are coupled through the requirements of element conservation, and the rate-controlled subsystem can be thought of as a source or sink of chemical species which forms a set of boundary conditions for the equilibrium subsystem. If the equilibrium subsystem is assumed to include species 1, . . ., M, the conditions $DG_i/Dt = 0$ yield M equations:

$$\frac{\partial G_k}{\partial \rho}\left(\rho \frac{D\rho}{Dt}\right) + \frac{\partial G_k}{\partial T}\left(\rho \frac{DT}{Dt}\right) = -\sum_{i=1}^{N}\left(\frac{\partial G_k}{\partial Y_i}\right)\left(\rho \frac{DY_i}{Dt}\right)$$

$$k = 1, \ldots, M \qquad (2)$$

as shown by Libby. The right-hand side of each equation will involve only Y_k, Y_{N-L+1}, . . ., Y_N. There will be N-L-M equations of the form of Eq. (1) remaining:

$$\rho(DY_i/Dt) = \dot{w}_i \quad i = M + 1, \ldots, N\text{-}L \qquad (3)$$

Since none of the σ_j, $j = 1, \ldots, M$ appear in these equations, the right-hand sides will be determinate. Together with the element conservation equations, these form a set of algebraic equations that may be solved for the unknown species derivatives at any point in the flow.

The reason that the mixing of equilibrium and nonequilibrium assumptions must be approached with such care is that the equilibrium limit of the nonequilibrium problem is a singularity. For true chemical equilibrium to exist, as required by the condition $G_j = 0$, there must be no net production or consumption of any chemical species. This is impossible in any system in which finite gradients are present, as in a reacting flow. In computer solutions of reacting flow problems, difficulties are encountered when the forward and reverse reaction rates both are very large and their difference is too small to compute accurately. In such cases the equilibrium assumption that both forward and reverse rates are infinitely large and equal may furnish a good approximation. However, the gradient in species concentration computed from Eq. (2) is not dependent on the known finite rates of any reactions involving species k. Rather it is the rate at which the species concentration must change in order to maintain equilibrium (zero net production) when the density and temperature are changing. Hence the analogy between equilibrium and nonequilibrium is not complete.

Author's Reply to Comment by R. E. Mates

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The present author assumed in the note under discussion[1] that the problem was "well-posed," that is, that the equilibrium conditions corresponding to the "fast" reactions, $G_j \simeq 0$, were indeed all independent; if they are not, the problem is "ill-posed" by definition. Thus the observations in the second paragraph do not appear appropriate.

In addition, the present author does not understand the first two sentences of the third paragraph of the foregoing comment. It is precisely in order "to compute contributions to gradients in species concentrations due to individual reactions," namely, from the "fast" reactions, and because the concentration of a species "involved in local chemical equilibrium with some other species" *cannot* "be determined without reference to the chemical kinetics" of the "slow" reactions that there exists a problem in applying the concept of partial equilibrium in flowing systems, a problem posed and solved by one of perhaps several methods by the present author.

The system described in connection with Eq. (2) of the preceding comment appears to be highly artificial in that the M fast reactions are postulated to involve only M species; in this case, the commentor is correct that no indeterminacies arise and that no problem exists.

With respect to the final paragraph, it is understood generally by fluid mechanicians that the concept of equilibrium chemical behavior in a flowing system involves whatever "net production or consumption of any chemical species" required to maintain at each point in the flow an equilibrium state. The forward and reverse rates corresponding to a single reaction step at equilibrium are relatively large *but not equal*. The difficulties connected with obtaining equilibrium flows from computer programs applicable to finite rate chemistry are well known and, of course, are associated with the size of the reaction rates.

In view of the doubts raised by Mates[1] concerning the present author's treatment of partial equilibrium, it may be appropriate to point out that the procedure in question has been employed to study the heat release phase of the hydrogen-air reaction carried out at constant pressure;[2] there was no difficulty encountered in applying the treatment under discussion.

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Errata: "The Status of Unsteady Aerodynamic Influence Coefficients"

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AND

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The derivation of aerodynamic influence coefficients (AICs) from slender-body theory given in Ref. 1, pp. 60–72, has been found to be incorrect. Since the standard NASA stability-axis system is used (with the relative wind in the *negative x* direction), the substantial derivative operator in Eq. (179) et seq. should read $-V(\partial/\partial x) + (\partial/\partial t)$, and, hence, the sign of the velocity should be reversed throughout the derivation. The net result is that the signs of the imaginary parts of all the AIC elements (i.e., the damping terms) are reversed; the signs of the real parts are not affected. The mistake was discovered while programming the equations for digital computation, and the corrected derivation as well as the FORTRAN computer program are reported in Ref. 2.

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